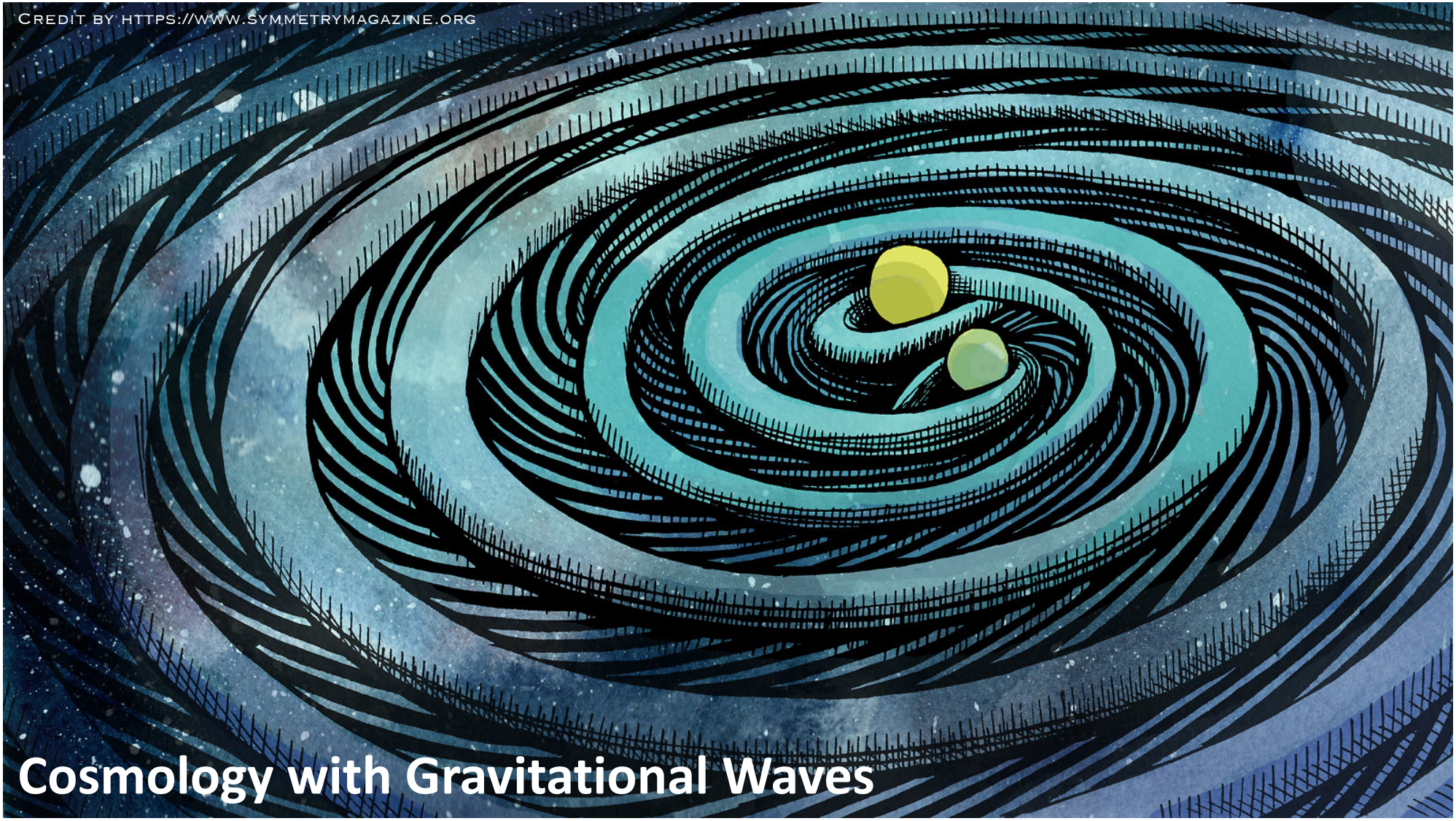

Cosmology with Gravitational Waves

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INFN
ICCUB
Excelencia María de Maeztu

Tunder and lightning

- Until a few years ago we have only seen the Universe (and 95% of it is dark: dark matter and dark energy)
- Now we finally are able to listen to the Universe.

This is revolutionary!

A revolutionary new and completely different way to study the Universe

Through the detection and observation of gravitational waves (GWS):

- Trace the formation, growth, and merger history of massive black holes
- Explore stellar populations and dynamics in galactic nuclei
- Test General Relativity with observations
- Probe new physics and cosmology
- Survey compact stellar-mass binaries and study the structure of the Galaxy

Properties of GWs

- Propagate at the speed of light (in “standard” GR)
- Transverse to the direction of propagation
- Two polarizations (+ and x)

$$h_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- Here we are assuming a weak gravitational field in GR where $g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}\,, \quad |h_{\alpha\beta}| \ll 1$
- GWs are propagating in the *z*-direction;
- GWs are propagating waves of spacetime curvature, tidally stretching and squeezing as they radiate from their source into the Universe.
-The corresponding line element is

$$ds^2 = -dt^2 + (1 + h_+)\,dx^2 + (1 - h_+)\,dy^2 + 2h_\times\,dx\,dy + dz^2$$

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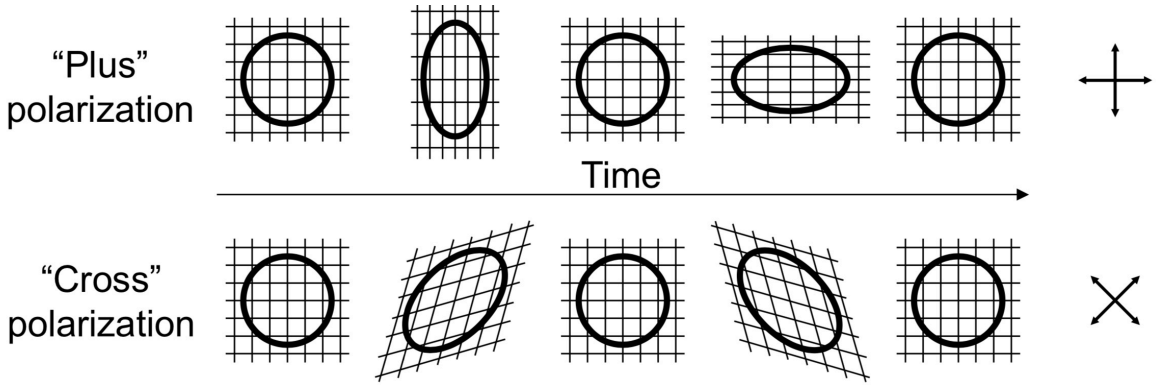
$$h_{\alpha\beta} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & -h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

GWs are deformations of space itself, stretching it first in one direction, then in the perpendicular direction.



Properties of GWs

- Propagate at the speed of light (in “standard” GR)
- Transverse to the direction of propagation
- Two polarizations (+ and x)
- Carry energy

The GWs a source emits backreact upon it, which appears as a loss of energy and angular momentum.

The “quadrupole formula” predicts that a system with a time changing quadrupole moment will lose energy to GWs according to

$$\frac{dE}{dt} = -\frac{1}{5}\frac{G}{c^5}\sum_{ij}\frac{d^3Q_{ij}}{dt^3}\frac{d^3Q_{ij}}{dt^3}$$

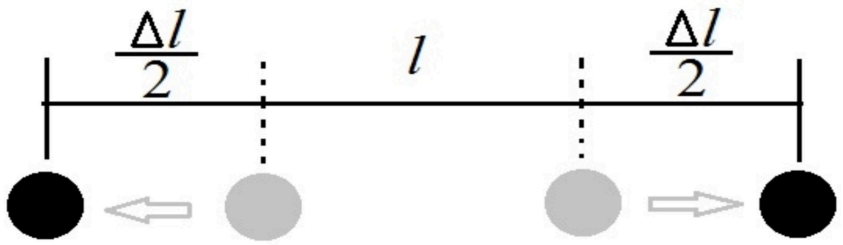
where

$$h_{ij} = \frac{2G}{c^4}\frac{1}{r}\frac{d^2Q_{ij}}{dt^2}$$

$$Q_{ij} = \int \rho\left(x_i x_j - \frac{1}{3}\delta_{ij}r^2\right)dV$$

Properties of GWs

- Propagate at the speed of light (in “standard” GR)
- Transverse to the direction of propagation
- Two polarizations (+ and x)
- Carry energy
- Affect the relative separation of test particles



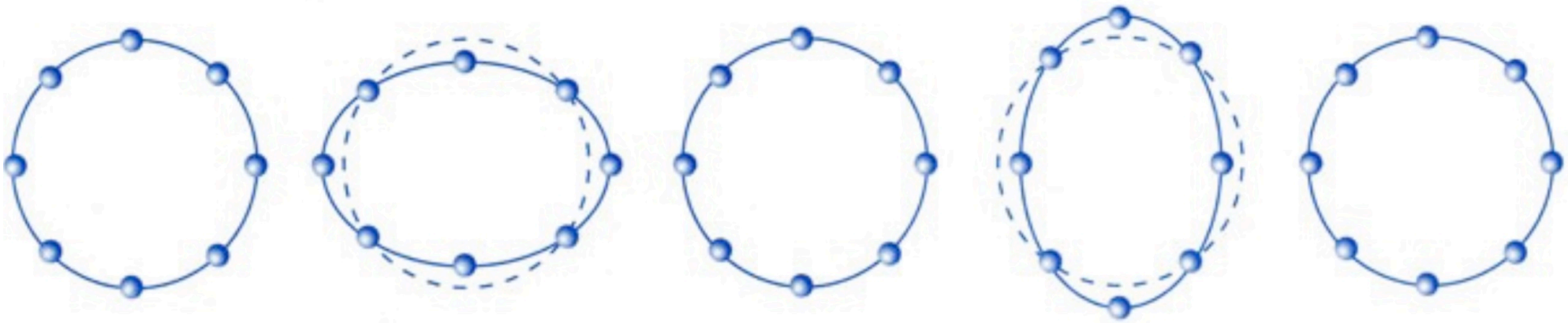
Δl is the change in the spacing between particles due to gravitational wave, l is the initial distance between particles, and h is the fractional change in distance (strain) and given by

$$h=\Delta L/L$$

Properties of GWs

In general

- h=ΔL/L is more complex and depends upon the **geometry of the measurement device**, the **arrival direction**, and the **frequency** and **polarization** of the GW
- h **stretch** and **shrink** the distance between two points



$$\frac{\Delta L}{L} = \frac{1}{2} h_{\mu\nu} n^{\mu} n^{\nu}$$

Properties of GWs

In general

- h=ΔL/L is more complex and depends upon the **geometry of the measurement device**, the **arrival direction**, and the **frequency** and **polarization** of the GW
- h **stretch** and **shrink** the distance between two points
- h is the magnitude of a typical component $h_{\alpha\beta}$

$$h \approx \frac{G}{c^4}\frac{mv^2}{r}$$

v is the typical speed associated with the source's quadrupolar dynamics, and *m* is proportional to the mass that participates in those dynamics.

h=ΔL/L ~ 10^{-22} **Gravitational waves are very very very weak!**

To produce strong GWs need large masses (e.g., at least the mass of the Sun) moving very fast (e.g., near the speed of light)

Properties of GWs

In general

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- h **stretch** and **shrink** the distance between two points
- h is the magnitude of a typical component $h_{\alpha\beta}$

$$h \approx \frac{G}{c^4}\frac{mv^2}{r}$$

$$\approx 10^{-22} \times \left(\frac{200\,\mathrm{Mpc}}{r}\right) \times \left(\frac{M}{3M_\odot}\right) \times \left(\frac{v}{0.3c}\right)^2$$

$$\approx 10^{-20} \times \left(\frac{6\,\mathrm{Gpc}}{r}\right) \times \left(\frac{M}{10^6\,M_\odot}\right) \times \left(\frac{v}{0.1c}\right)^2$$

Numbers characterizes stellar mass sources that are targets for ground-based high- frequency detectors

Numbers characterizes massive black holes that are targets of space-based low-frequency detectors

Properties of GWs

In general

- h=ΔL/L is more complex and depends upon the **geometry of the measurement device**, the **arrival direction**, and the **frequency** and **polarization** of the GW
- h **stretch** and **shrink** the distance between two points
- h is the magnitude of a typical component $h_{\alpha\beta}$
- h depends on the kind of wave to be detected and this in turn depends on how the wave was produced and how far its source is from an observer!

The Gravitational Wave Spectrum

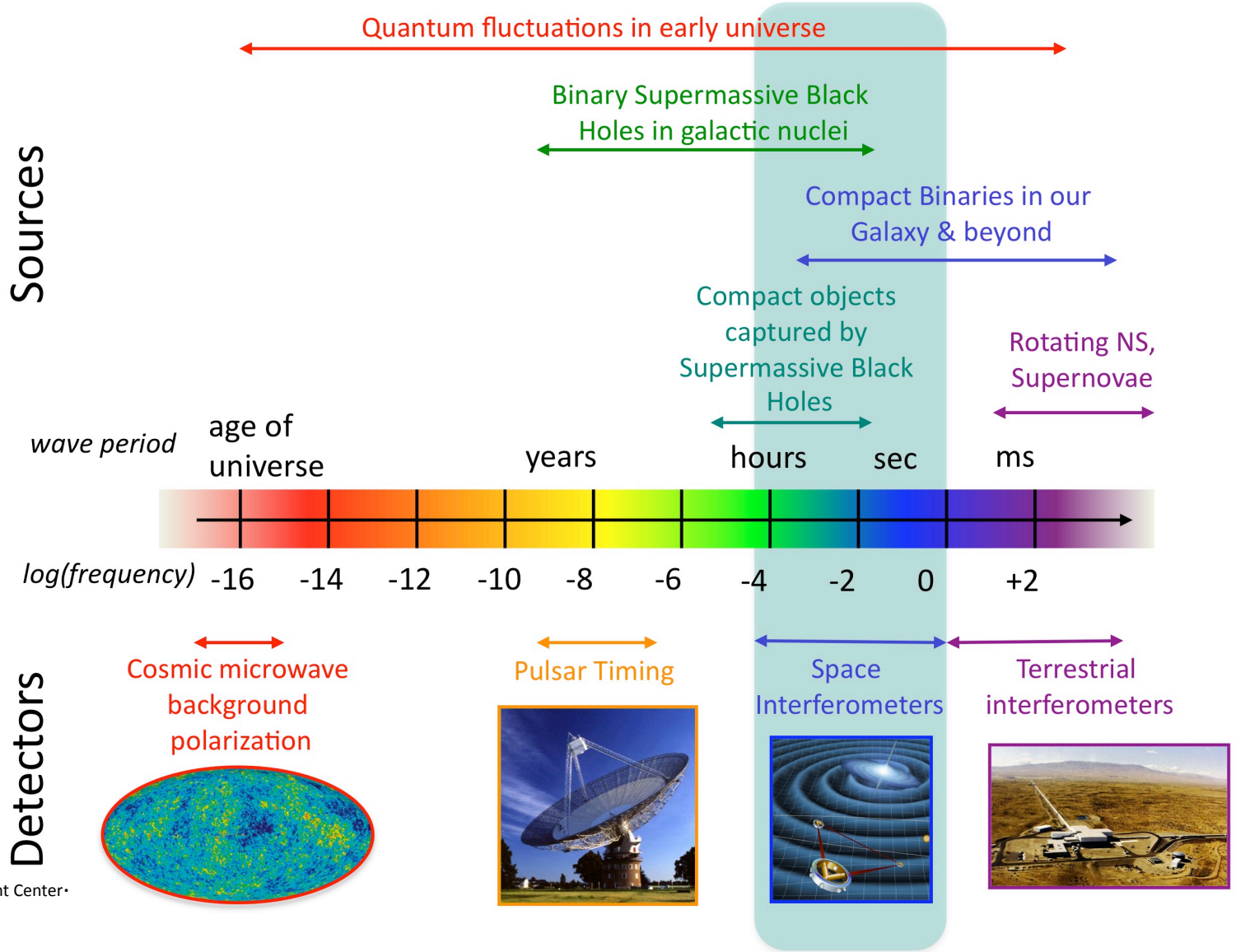


Figure credit:
NASA Goddard Space Flight Center

GWs radiate at a frequency inversely proportional to their mass. Such sources are more intense and are expected to have higher amplitudes.

Gravitational waves are classified into three types:

- Impulsive (bursts):

Intense gravitational radiation

→ produced by a supernova explosion

→ gamma ray bursters

→ through the merging of two black holes.

- Periodic:

corresponds to those whose frequency is more or less constant for long periods of time

→ Gws may have their origin in binary neutron stars rotating around their center of mass,

→ From a neutron star that is close to absorb material from another star (accreting neutron star)

- Stochastic:

Stochastic waves contribute to the gravitational background noise

→ possibly have their origin in the Big Bang.

→ stochastic backgrounds due to Black Hole-Black Hole coalescences.

Three different wave types appear in different parts of the spectrum

High frequency

- The high-frequency band of roughly 1-1000 Hz is targeted by ground-based laser interferometers.

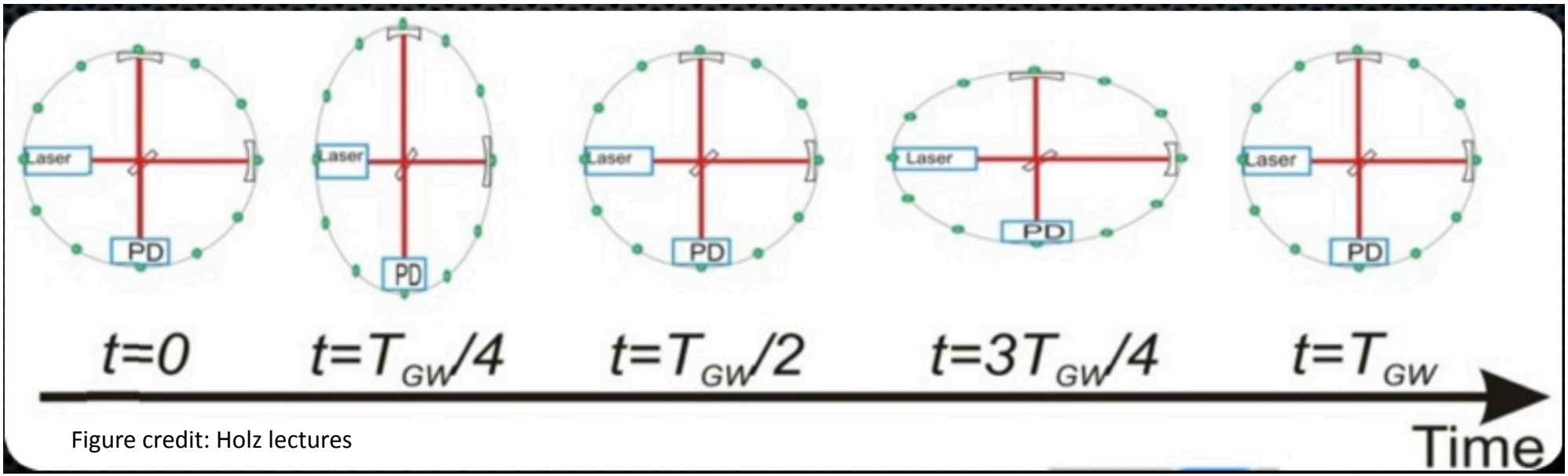
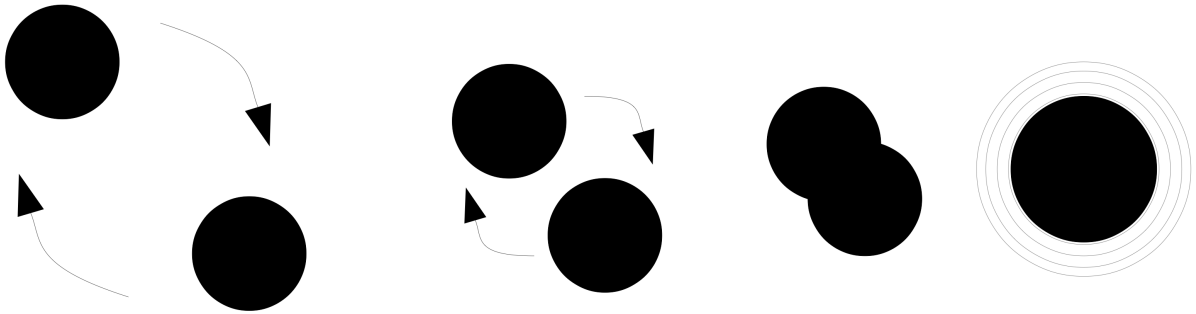


Figure credit: Holz lectures

If the distance between one of the mirrors to the light splitter varies by an amount Δl with respect to distance to the same splitter of the second mirror, then the recombined beam will change its intensity. From measuring the intensity change of the recombined light beam, it is possible to obtain Δl.

High frequency

- The high-frequency band of roughly 1-1000 Hz is targeted by ground-based laser interferometers.
- The expected sources need to be compact (e.g., neutron stars or black holes) and the inspiral needs to be in its final stages (last few minutes) in order for the GWs to be detectable by Earth-based interferometers.



- For such sources, the natural GW frequency is in the high-frequency band if $M \sim 1 - 100\ M_{\odot}$
- The most prominent facilities are those of LIGO in the USA, VIRGO in Italy, GEO600 in Germany, and KAGRA in Japan, which are all running or expected to run soon.
- Finally with the third-generation ground-based interferometer Einstein Telescope, it will be possible to reach frequencies of the order of 1 Hz (since going underground helps to partially overcome the seismic noise).

Low frequency 10^{-5} Hz <f< 0.1 Hz

- The expected sources are merging of very massive Black Holes at high redshifts
- It should also detect waves from tens of stellar-mass compact objects spiraling into onto strong-field orbits of central massive ~ 10^6 $M\odot$ Black Holes [i.e. EMRIs Extreme Mass Ratio Inspirals (EMRIs)]
- Binary stars and binary white-dwarf systems formed in our Milky Way,
- It is targeted using laser interferometry between spacecraft.

Low frequency 10^{-5} Hz <f< 0.1 Hz

The advantage of a space-based GW interferometer resides in its capability to reach high sensitivity in the intermediate frequency band below 1 Hz.

- In 2013, ESA has approved a GW observer in space as the L3 mission for launch in 2034, for which the “evolved LISA” (eLISA) space-based interferometer is the main candidate.
- eLISA plans to detect gravitational waves by measuring separation changes between ducial masses in three spacecrafts that are supposed to be 5 million kilometers apart!

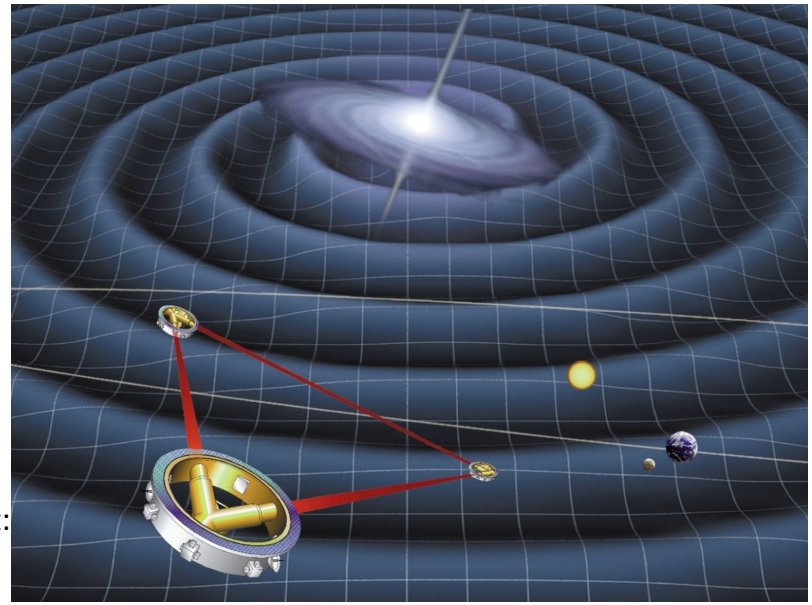
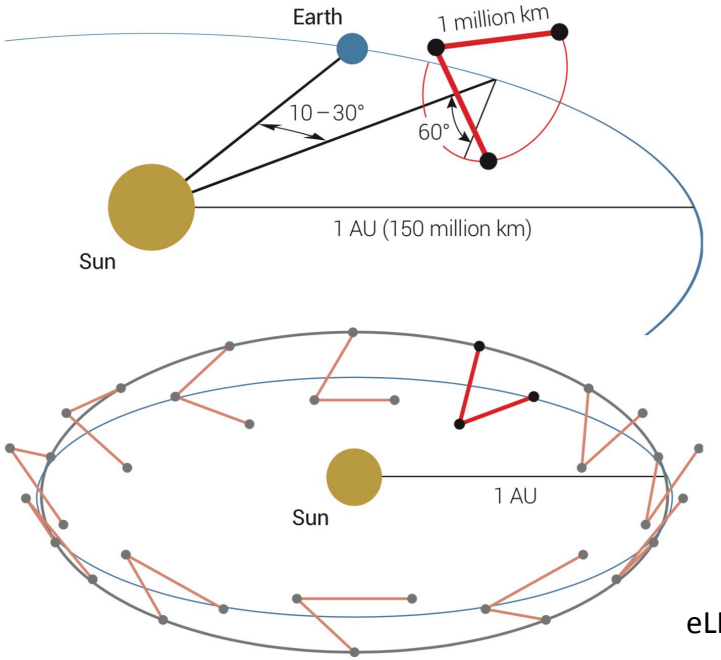


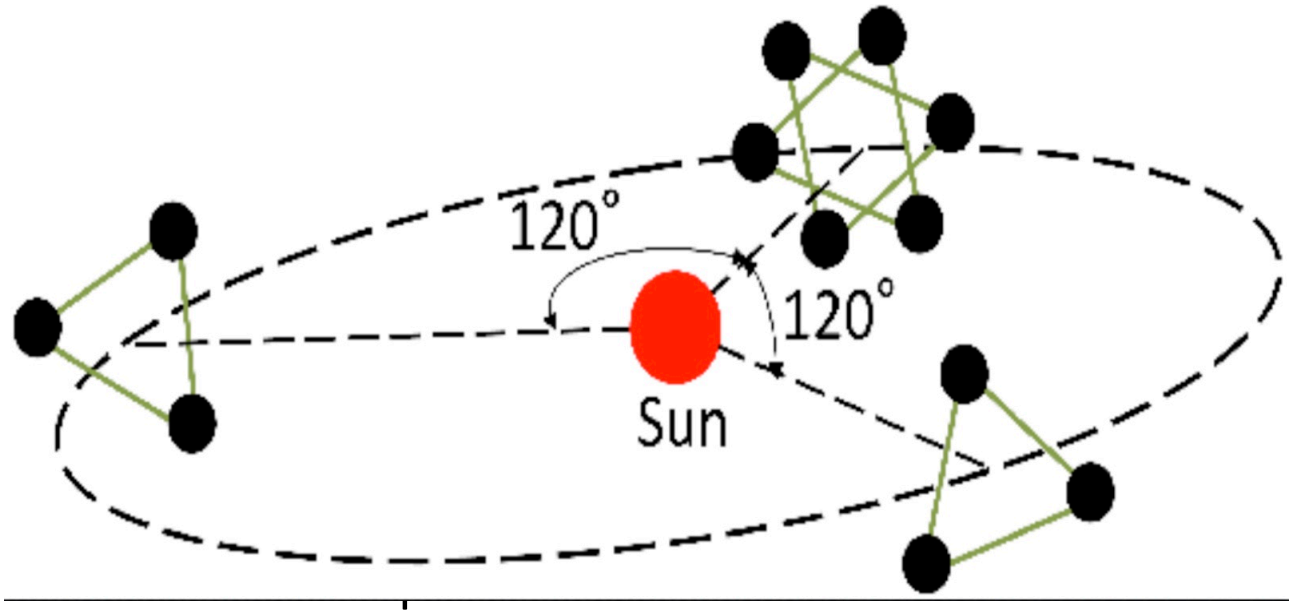
Figure credit: LISA



eLISA Orbits

Low frequency

10^{-5} Hz <f< 0.1 Hz



The configuration of DECIGO and BBO. There are 8 effective interferometers in total.
BBO consists of four LISA-like triangular constellations orbiting the Sun at 1 AU
Decigo would be almost identical, except that the constellations are 50 times smaller than BBO

- Future space-based gravitational-wave (GW) detectors such as DECI-hertz Interferometer Gravitational-wave Observatory (DECIGO) and Big-Bang Observer (BBO) are the most sensitive to GWs in 0.1 − 1 Hz band
- DECIGO and BBO will aim at detecting the primordial GW background, the mergers of intermediate-mass black holes (BH), and a large number (~ 10^6) of neutron-star (NS) binaries in an inspiraling phase.

DECIGO and BBO will provide a novel opportunity to measure the property of the Universe!

Very low frequency $10^{-9} - 10^{-6}$ Hz

- In this frequency band, the two most plausible sources are

  1) the coalescence of massive binary black holes

  [Population synthesis estimates based on models of structure formation and galaxy growth suggest there should be a substantial population of such binaries whose members are black holes of $10^6 - 10^8\ M_\odot$]

  2) a high-frequency tail of the primordial GWs

  The GWs produced by these binaries combine to form a stochastic background in the very low frequency band.

- This background is targeted by pulsar timing observations:

  This technique uses the fact that millisecond pulsars are very precise clocks; indeed, the stability of some pulsars rivals laboratory atomic clocks.

- Three Pulsar Timing Arrays are currently in operation – the EPTA in Europe, NANOGrav in the US and PPTA in Australia – sharing data under the aegis of the International Pulsar Timing Array (IPTA)

Ultra low frequency 10^{-13} Hz< f< 10^{-18} Hz

- This low frequency GW band is best described using wave-length: it consists of GWs with $c/H0>\lambda>10^{-5}c/H0$. These are waves that vary on lengthscales comparable to the size of our Universe!
- Following inflation, the GWs that are produced by this process propagate through the Universe.
- GW barely interact with matter as they propagate, just stretching and squeezing the primordial plasma in the young expanding Universe.
-This stretching and squeezing creates a quadrupolar temperature anisotropy in the plasma at recombination, which causes the CMB to be linearly polarized. The GWs thus leave an imprint on the CMB.
- Then we expect primordial gravitational waves stemming from the inflationary era of the very early Universe. Primordial quantum fields fluctuate and yield space–time ripples at a wide range of frequencies. These could in principle be detected as B-mode polarization patterns in the CMB radiation, at large angles in the sky.

Gravitational Universe

The beauty of General Relativity (GR) is that it is a falsifiable theory

Thus even a single experiment incompatible with a prediction of the theory would lead to its invalidation

(For example) To test GR:

- in strong gravity regime
- in cosmology

Tests of strong gravity and non-linear regime

- The strong- field realm of gravity theories can be probed:

  1) near the event horizon of Kerr black holes

  2) in other large-curvature environments (e.g., in the early Universe)

For example:

- Two black holes (or neutron stars) crash into each other
- A star falls into a big black hole, i.e.

  EMRIs Extreme Mass Ratio Inspirals (EMRIs)
- A supernova explodes (asymmetrically)
- Big bang/inflation

Tests of strong gravity and non-linear regime

Leaving open several questions:

- *Does gravity travel at the speed of light ?* **YES! From GW170817**
- *Does the graviton have mass?*
- *How does gravitational information propagate: Are there more than two transverse modes of propagation?*
- *Does gravity couple to other dynamical fields, such as, massless or massive scalars?*
- *What is the structure of spacetime just outside astrophysical black holes? Do their spacetimes have horizons?*
- *Are astrophysical black holes fully described by the Kerr metric, as predicted by General Relativity?*

To answer these questions and learn about the fundamental nature of gravity is by observing the vibrations of the fabric of spacetime itself.

Stochastic Gravitational Wave Background (SGWB)

Incoherent superposition of many unresolved sources.

- Cosmological:
  - » Inflationary epoch, preheating, reheating
  - » Phase transitions
  - » Cosmic strings
  - » Alternative cosmologies
- Astrophysical:
  - » Supernovae
  - » Magnetars
  - » Binary black holes
- Potentially could probe physics of the very-early Universe.

GW standard sirens

- GWs provide a unique, purely gravitational way to measure distance
- As GW detections can be thought of as aural rather than optical (Hughes 2003), a more appropriate term for a GW standard candle is a “standard siren”
- BH binary mergers as "standard sirens" to extract information on the expansion of the Universe, by measuring the expansion history with completely different techniques to electromagnetic probes.

GW standard sirens

- GWs provide a unique, purely gravitational way to measure distance
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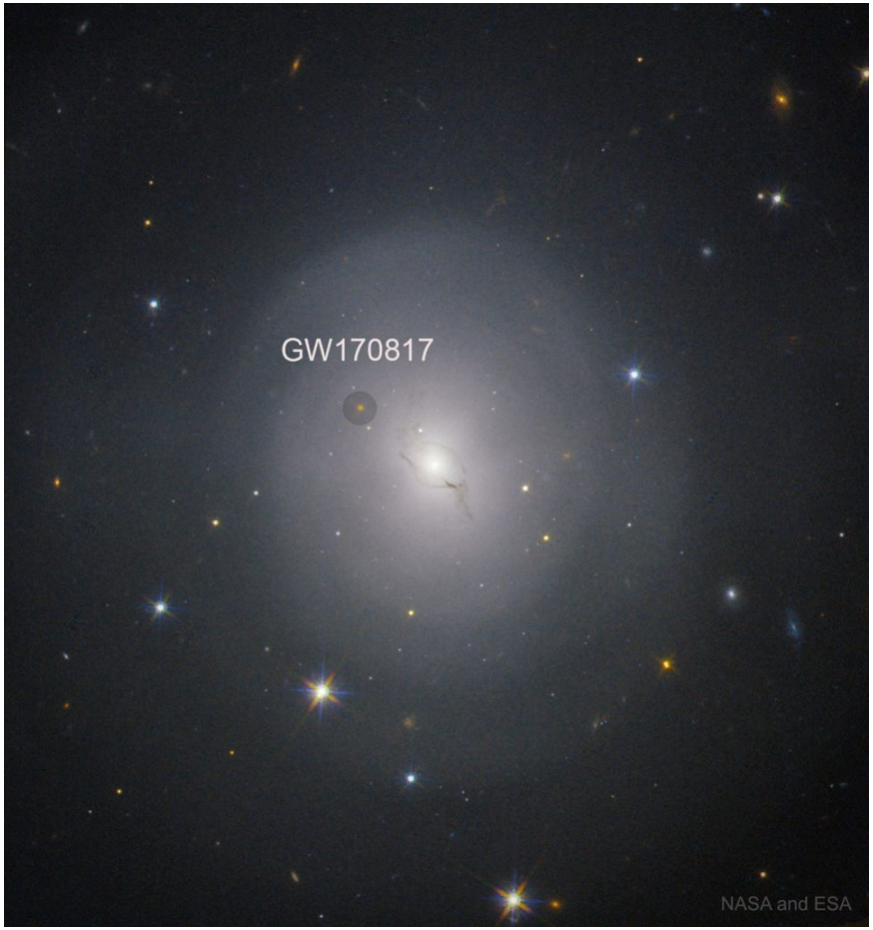
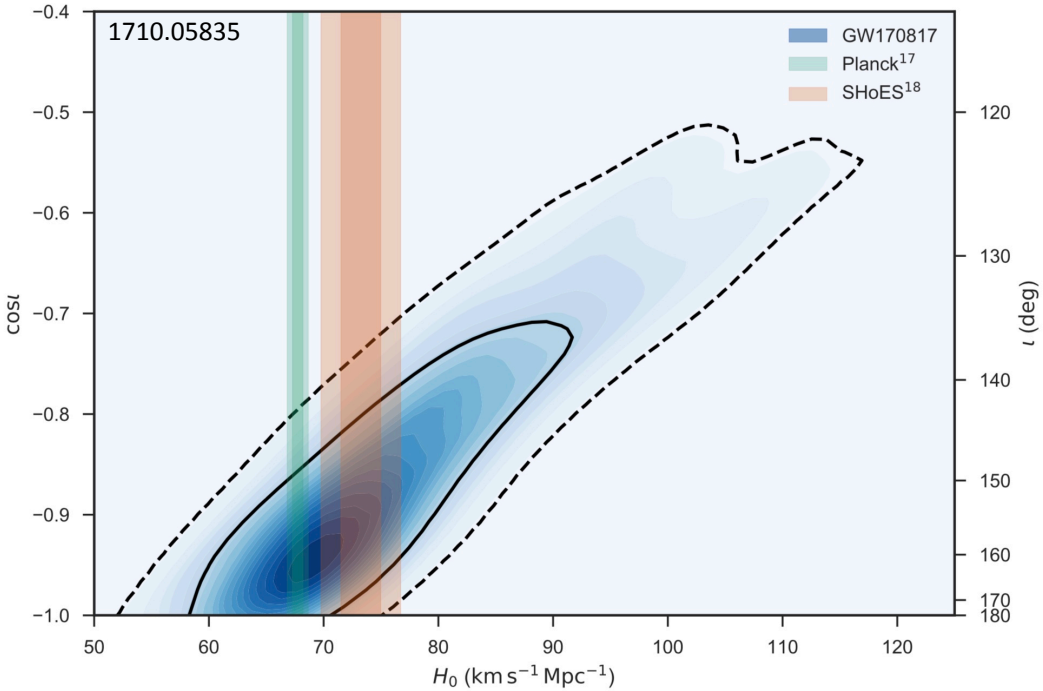
- BH binary mergers as "standard sirens" to extract information on the expansion of the Universe, by measuring the expansion history with completely different techniques to electromagnetic probes.
- Black hole coalescences could serve as standard sirens for cosmography by providing absolute and direct measurements of the luminosity distance D_L
- One advantage of standard sirens over SNIa is that they allow for a direct measurement of D_L up to large redshift, unlike optical measurements, which require cross-calibrations of successive distance indicators at different scales;

GW standard sirens





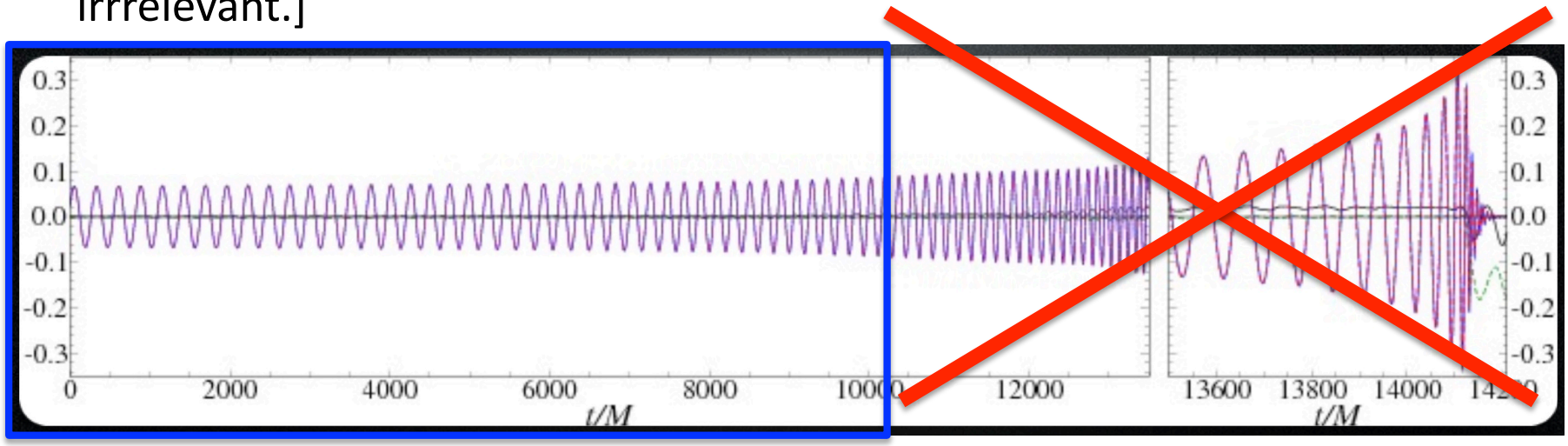
Posterior density of H0 and cos ι from the joint GW-EM analysis (blue contours). Shading levels are drawn at every 5% credible level, with the 68.3% (1σ, solid) and 95.4% (2σ, dashed) contours in black. Values of H_0 and 1- and 2σ error bands are also displayed from Planck (Planck Collaboration et al. 2016) and SHoES (Riess et al. 2016).

- This method has recently been used for the first (albeit still low-precision) gravitational measurement of the Hubble constant, using LIGO/VIRGO binary neutron star event, GW 170817.

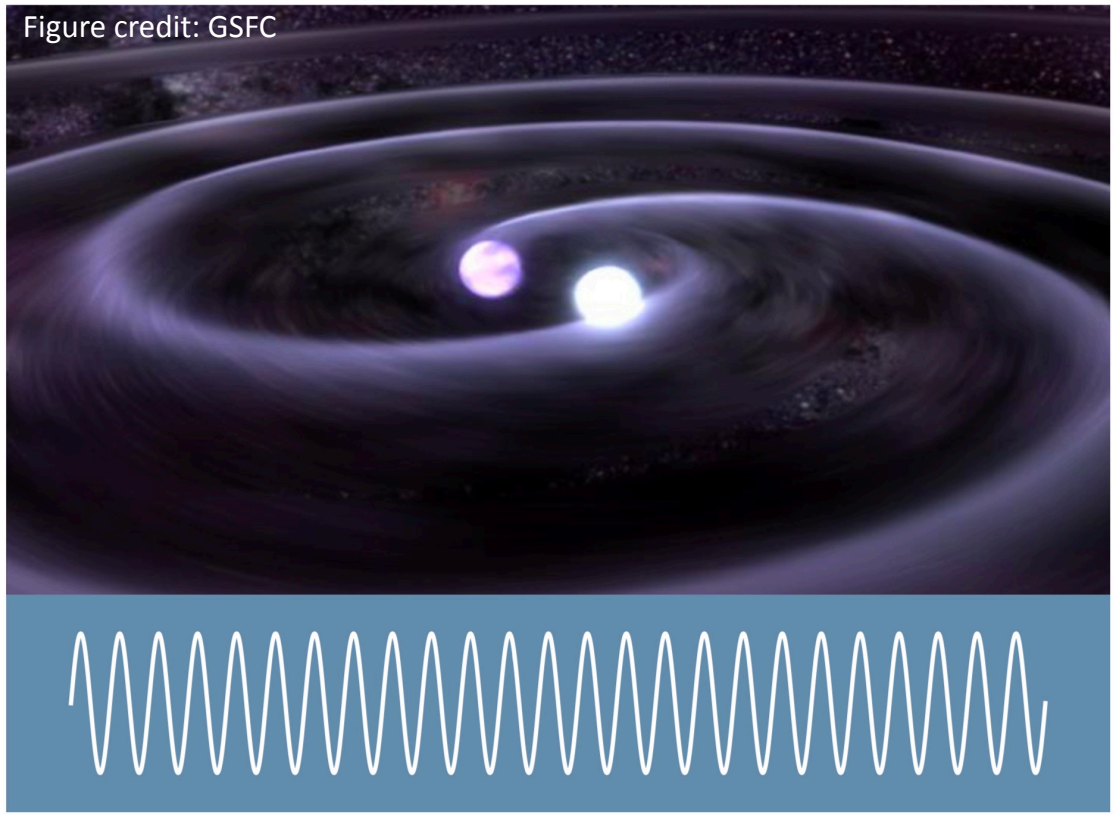
Cosmology with GW measurements

- Binary black hole inspirals are wll understood, theoretically clear and well-modelled (see Schutz 1986; Holz & Hughes 2005; Dalal, Holz, Hughes & Jain 2006; Cutler and Holz 2009; Nissanke et al. 2010, 2013)
- For distance determination the most interesting epoch of BH-BH coalescence is the inspiral, when the binary's members are widely separated and slowly spiral together due to back-reaction from GW emission.

  [The merger (in which the holes come into contact, forming a single body) and the ring- down (the final, simple stage of the ringdown, in which the merged binary is well-modeled as a single, distorted black hole) are irrelevant.]



Cosmology with GW measurements



- Binary is a nearly perfect quadrupole radiator and, unless general relativity fails in the deep strong field, its waves have a form that depends only on physical parameters of the system.
- The waveform depends most strongly on the source binary's masses and spins, the angles which determine its position on the sky and orientation with respect to the line of sight, and the distance to the source.

GWs from binary system

Schematically, a measured binary waveform takes the form

$$h_+ = \frac{2\mathcal{M}_z^{5/3}[\pi f(t)]^{2/3}}{D_L}\left[1+(\hat{L}\cdot\hat{n})^2\right]\cos[\Phi(t)]\ ,$$

$$h_\times = \frac{4\mathcal{M}_z^{5/3}[\pi f(t)]^{2/3}(\hat{L}\cdot\hat{n})}{D_L}\sin[\Phi(t)]\ .$$

- luminosity distance D_L
- accumulated GW phase $\Phi\left(m_1, m_2, \mathbf{S}_1, \mathbf{S}_2, t\right)$
- GW frequency $f(t) = (1/2\pi)d\Phi/dt$
- $\hat{n}$ points from the center of the barycenter frame to the system, and hence defines its position on the sky;
- $\hat{L}$ points along the binary's orbital angular momentum, and hence defines its orientation.
- Redshifted chirp mass $\mathcal{M}_z = (1+z)(m_1 m_2)^{3/5}/(m_1+m_2)^{1/5}$

Distance, but not redshift!

- Gravitational waves provide a direct measure of luminosity distance, but they give no independent information about redshift
- GWs from a local binary with masses (m_1, m_2)
- at redshift z is indistinguishible from a local binary with masses [$m_1/(1 + z)$, $m_2/(1 + z)$]
- In general to measure cosmology, need independent measurement of redshift

Approaches to standard siren cosmology

- **To determine cosmological parameters (e.g. H_0)**

- **To establish luminosity distance-redshift relation**

- Using the electromagnetic (EM) counterpart

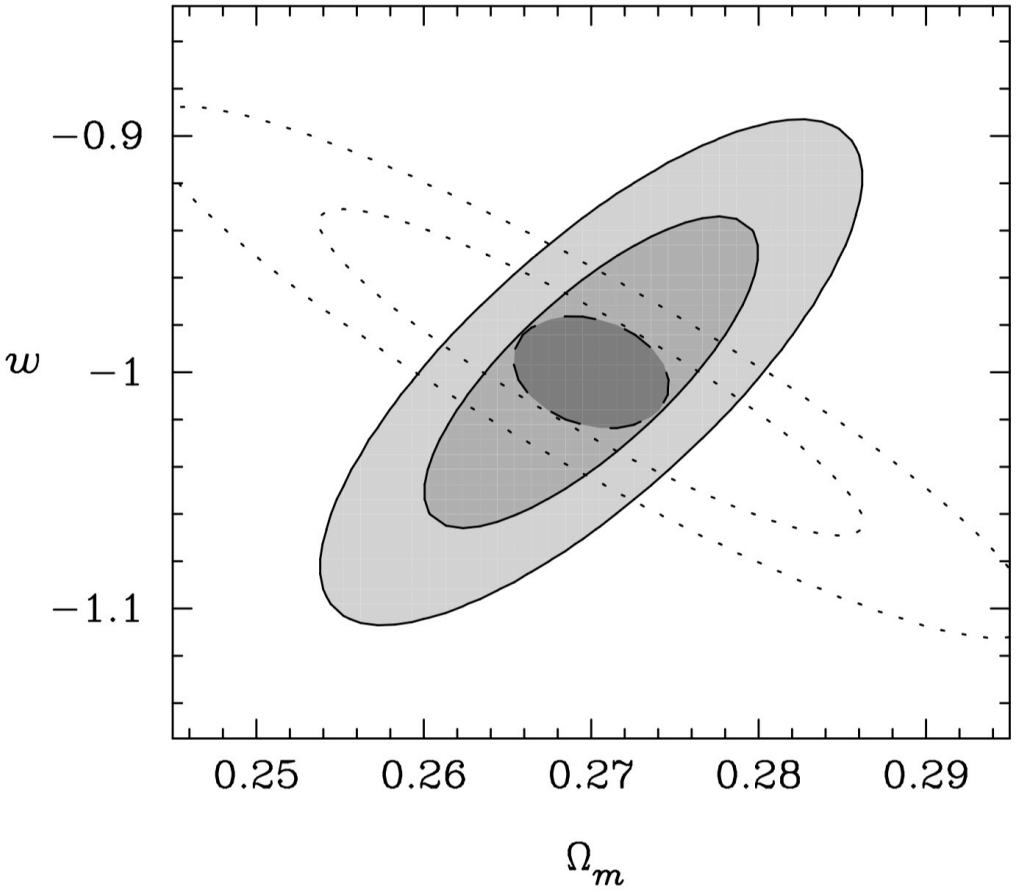
- Using a statistical solution-method (absence of EM counterparts)

Approaches to standard siren cosmology

- EM counterpart case, electromagnetic observations identify a counterpart to the GW source.
- This can be done by directly observing a transient electromagnetic source, such as a gamma-ray burst/afterglow or a kilonova.
- The redshift can then be determined, either directly from the counterpart, or by identifying the host galaxy associated with the counterpart and measuring its redshift instead.

Approaches to standard siren cosmology

- LISA constraints on dark energy equation of state (Dalal et al 2006)



100 SMBBH sources observed by LISA, distributed with constant comoving density between $0 < z < 2$

The dotted contours correspond to a sample of 3000 SNe with intrinsic luminosity scatter of 10%, with redshift distribution over $0.02 < z < 2$

The dashed (dark shaded) contour shows the 68% confidence region for the combined constraints GW+SNe+CMB.

Approaches to standard siren cosmology

- A statistical solution-method (absence of EM counterparts) assumes a source redshift distribution based on galaxy catalogs [MacLeod and C. J. Hogan 2008; Petiteau et al 2011].
- Since each GW event typically has a large error volume on the sky, then by combining a large number of such sources, a value of the cosmic expansion rate can be obtained that maximizes the likelihood fit.
- Such a method, however, colud be limited to redshifts $z < 1\text{-}2$ where galaxy catalogs are complete.

Approaches to standard siren cosmology

- A third method is to build up a sufficient catalog of GW events that the anisotropies in the distribution of events on the sky can be matched to the known clustering of large scale structure [Namikawa, Nishizawa & Taruya, (2016a, b); Oguri (2016)].
- Essentially, one cross-correlates galaxies with gravitational-wave standard sirens in order to maximize the likelihood of cosmological parameters, and thereby determine the distance-redshift relationship.

Cosmological perturbation effects on GW luminosity distance estimates

- In DB, Raccanelli, Bartolo & Matarrese (1702.01750), we consider the observer frame as reference system (the usual calculation is performed in a homogeneous and isotropic FLRW Universe, see Laguna et al 2010);
- We consider the local wave zone approximation to define the tetrads at source position:

  [Note that we are assuming that the observer “at the emitted position” is within a region with a comoving distance to the source sufficiently large so that the gravitational field is “weak enough” but still “local”, i.e. the gravitational wave wavelength is small w.r.t. the comoving distance from the observer]
- We assume the Newtonian approximation which agrees with standard weak-field approximation in general relativity (i.e., for simplicity, we are neglecting the post newtonian terms)
- We consider only the regime called of “quasi- circular” motion (i.e. the approximation in which a slowly varying orbital radius is applicable)

Cosmological perturbation effects on GW luminosity distance estimates

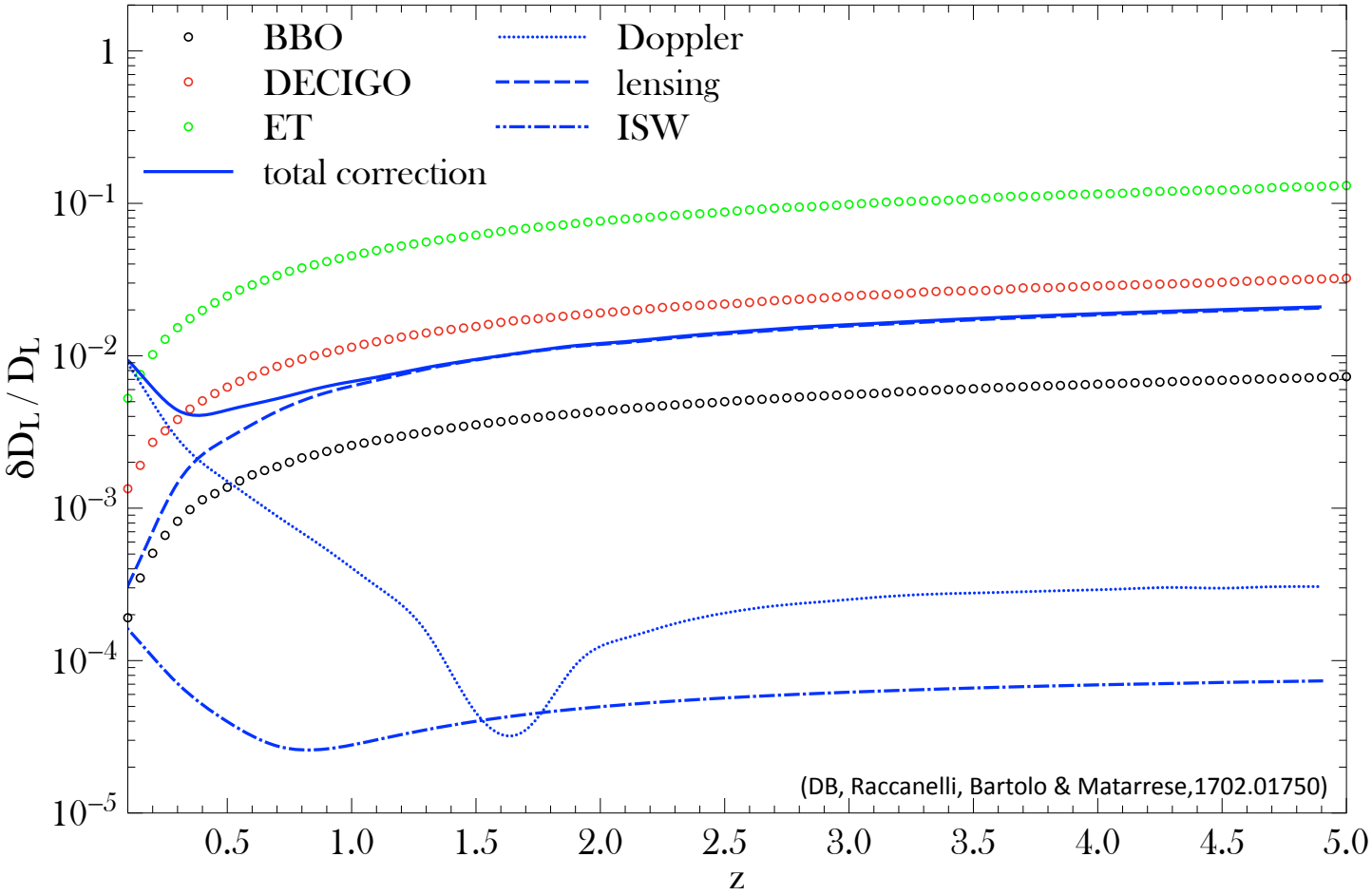
Computing the modifications of the value of the luminosity density D_L inferred from gravitational waves, due to perturbations, we can write the correction to the luminosity distance as

$$\frac{\Delta\mathcal{D}_L}{\bar{\mathcal{D}}_L} = \left(1-\frac{1}{\mathcal{H}\bar{\chi}}\right)v_\parallel - \frac{1}{2}\int_0^{\bar{\chi}} \mathrm{d}\tilde{\chi}\frac{(\bar{\chi}-\tilde{\chi})}{\tilde{\chi}\bar{\chi}}\ \triangle_\Omega\left(\Phi+\Psi\right) +$$
$$+\ \frac{1}{\mathcal{H}\bar{\chi}}\Phi - \left(1-\frac{1}{\mathcal{H}\bar{\chi}}\right)\int_0^{\bar{\chi}}\mathrm{d}\tilde{\chi}\left(\Psi'+\Phi'\right) - \left(\Phi+\Psi\right) + \frac{1}{\bar{\chi}}\int_0^{\bar{\chi}}\mathrm{d}\tilde{\chi}\left(\Phi+\Psi\right)$$

where $\triangle_\Omega \equiv \bar{\chi}^2\bar{\nabla}^2_\perp = \bar{\chi}^2(\bar{\nabla}^2 - \bar{\partial}^2_\parallel - 2\bar{\chi}^{-1}\bar{\partial}_\parallel) = (\cot\partial_\theta + \partial^2_\theta + \partial_\varphi/\sin^2\theta)$

We can recognize the presence of a velocity term (the first r.h.s. term), followed by a lensing contribution, and the final four terms account for SW, ISW, volume and Shapiro time-delay effects.

Total correction to luminosity distance estimates due to perturbations



Points show the predicted precision in measurements of the luminosity distance, at any redshift, for the Einstein Telescope (green points) [Taylor et al 2012], DECIGO (red points) and the Big Bang Observer (black points) [Camera & Nishizawa 2013]

In the the next few years we will finally be able to listen to the Universe.

This will be revolutionary!