Parity breaking in gravitational Chern-Simons bispectra

Giorgio Orlando

Universitá degli studi di Padova, Dipartimento di Fisica e Astronomia "Galileo Galilei"

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Outline

- Introduction to Chern-Simons gravity during inflation
- Parity breaking signatures in primordial bispectra
- Observations through CMB bispectra
- Summary and Conclusion

- Presentation based on:

- *N. Bartolo, G. Orlando, JCAP07(2017)034*
- *M. Shiraishi, G. Orlando, N. Bartolo*, in preparation

Chern-Simons gravitational term coupled to the scalar sector during inflation

$$\mathcal{L} = \sqrt{g}\left[\frac{1}{2}M_{Pl}^2 R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right] + f(\phi)\epsilon^{\mu\nu\rho\sigma}C_{\mu\nu}{}^{\kappa\lambda}C_{\rho\sigma\kappa\lambda}\,.$$

Peculiarities of the Chern-Simons term:

- Parity breaking.
- Vanishing on the background ($C^{(0)}_{\mu\nu\kappa\lambda} = 0$).
- Only correlators involving gravitational waves are interesting.
- Invariant under Weyl transformation $g' = e^{-2w(x,t)}g$.

Chirality of gravitational waves

- GW become chiral: different time evolution for R and L states.

$$\gamma_R = \frac{1}{\sqrt{2}}(\gamma_+ - i\,\gamma_\times)\,, \qquad \gamma_L = \frac{1}{\sqrt{2}}(\gamma_+ + i\,\gamma_\times)\,.$$

Quadratic action of GW

$$S|_{\gamma\gamma} = \sum_{s=L,R} \int d\tau\, \frac{d^3k}{(2\pi)^3}\, A^2_{\gamma,s} \left[\, |\gamma_s'(\tau,k)|^2 - k^2 |\gamma_s(\tau,k)|^2 \,\right] ,$$

$$A^2_{\gamma,s} = \frac{M^2_{Pl}}{2} a^2 \left(1 - \lambda_s \frac{k_{phys}}{M_{CS}}\right) , \qquad M_{CS} = \frac{M^2_{Pl}}{8\dot{f}(\phi)}\,.$$

- $\lambda_R = +1$, $\lambda_L = -1 \Longrightarrow$ When $k_{phys} > M_{CS}$, γ_R becomes ghost field.
- SOLUTION : UV energy cut-off.

Killing the power-spectrum chirality

$$\mu_s = A_{\gamma,s}\gamma_s \, .$$

Equation of motion

$$\mu_s'' + \left(k^2 - \frac{\nu_T^2 - \frac{1}{4}}{\tau^2} + \lambda_s \frac{k}{\tau}\frac{H}{M_{CS}}\right)\mu_s = 0 \, .$$

- This equation works well only when $M_{CS} \simeq$ const.
- It is Whittaker equation, it can be solved analytically.

Super-horizon power-spectra

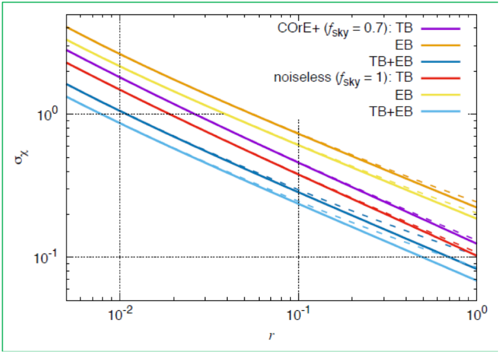
$$\Delta_\gamma^L = \frac{\Delta_\gamma}{2} e^{-\frac{\pi}{4}H/M_{CS}} \, ,$$
$$\Delta_\gamma^R = \frac{\Delta_\gamma}{2} e^{+\frac{\pi}{4}H/M_{CS}} \, .$$

$$\chi = \frac{\Delta_\gamma^R - \Delta_\gamma^L}{\Delta_\gamma^R + \Delta_\gamma^L} = \frac{\pi}{2}\frac{H}{M_{CS}} \, .$$

- $H/M_{CS} \ll 1$ due to the UV cut-off.

Forecasts on power-spectrum chirality from CMB data

- Only *TB* and *EB* correlators are sensitive to PS chirality.



M. Gerbino et. al. (2016)

- Impossible to observe small chirality given the current experimental constraint on r, $r < 0.07$ (95%CL).

Importance of considering higher order statistics

- The effects of Chern-Simons gravity on power-spectrum statistics can not be "measured" by CMB power spectra.
- This motivates an analysis of higher-order correlators.
- In JCAP07(2017)034 *"Parity breaking signatures from a Chern-Simons coupling during inflation: the case of non-Gaussian gravitational waves"* N. Bartolo, G. Orlando, we treated the bispectrum statistics of the model.
- We employed the in-in formalism formula (interaction picture):

$$\langle \delta_a(\vec{k_1})\delta_b(\vec{k_2})\delta_c(\vec{k_3})\rangle(t) = -i\int_{t_0}^{t} dt' \langle 0|\left[\delta_a^I(\vec{k_1},t)\delta_b^I(\vec{k_2},t)\delta_c^I(\vec{k_3},t)\ ,\ H_{int}^I(t')\right]|0\rangle\ .$$

Main results

- Amplitudes of $\langle\gamma\gamma\gamma\rangle$ and $\langle\gamma\zeta\zeta\rangle$ bispectra are suppressed by the ratio H/M_{CS} as the power spectrum case.
- Amplitude of $\langle\gamma\gamma\zeta\rangle$ bispectrum shows a different behaviour:

$$\langle\gamma_R(\vec{k}_1)\gamma_R(\vec{k}_2)\zeta(\vec{k}_3)\rangle = -(2\pi)^3\delta^3(\vec{k}_1+\vec{k}_2+\vec{k}_3)\frac{\pi}{64}\left(\sum_{i\neq j}\Delta_T(k_i)\Delta_T(k_j)\right)\times$$
$$\times\left(H^2\frac{\partial^2 f(\phi)}{\partial^2\phi}\right)\frac{(k_1+k_2)k_1k_2}{\sum_i k_i^3}\cos\theta(1-\cos\theta)^2\,,$$

$$\langle\gamma_L(\vec{k}_1)\gamma_L(\vec{k}_2)\zeta(\vec{k}_3)\rangle = -\langle\gamma_R(\vec{k}_1)\gamma_R(\vec{k}_2)\zeta(\vec{k}_3)\rangle\,,\qquad \langle\gamma_L(\vec{k}_1)\gamma_R(\vec{k}_2)\zeta(\vec{k}_3)\rangle = 0\,.$$

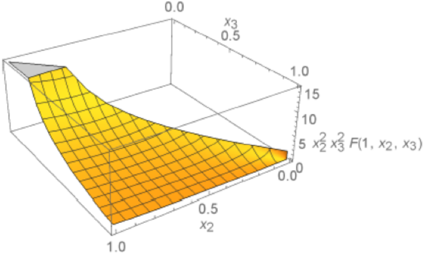
- The amplitude of this bispectrum is proportional to the second order derivative of $f(\phi)$. It does not appear suppressed as the power spectrum case.

Shape function

- The shape function gives the dependence of the bispectrum from the three momenta k_i.

$$F(k_1^\gamma, k_2^\gamma, k_3^\zeta) = \left(\sum_{i\neq j} \frac{1}{k_i^3 k_j^3}\right) \frac{(k_1+k_2)k_1 k_2}{\sum_i k_i^3} \cos\theta(1-\cos\theta)^2,$$

$\cos\theta = \frac{k_3^2 - k_2^2 - k_1^2}{2k_1 k_2}$.



$$x_2 = k_2/k_1\,, \quad x_3 = k_3/k_1.$$

- The shape function is maximum in the squeezed limit when the momentum of ζ is much smaller than the momenta of the two γ's ($k_3 \ll k_1 \simeq k_2$).

Chirality in $\langle\gamma\gamma\zeta\rangle$ bispectrum

- Parity breaking coefficient:

$$\Pi = \frac{\langle\gamma_R(\vec{k})\gamma_R(\vec{k})\zeta(\vec{k})\rangle_{TOT} - \langle\gamma_L(-\vec{k})\gamma_L(-\vec{k})\zeta(-\vec{k})\rangle_{TOT}}{\langle\gamma_R(\vec{k})\gamma_R(\vec{k})\zeta(\vec{k})\rangle_{TOT} + \langle\gamma_L(-\vec{k})\gamma_L(-\vec{k})\zeta(-\vec{k})\rangle_{TOT}},$$

$$\Pi = \frac{96}{25}\pi\left(H^2\frac{\partial^2 f(\phi)}{\partial^2\phi}\right).$$

- We can link this parameter to CMB bispectrum statistics...

Measurement of Π using CMB bispectra

- Link to the *BBT* angular bispectrum:

$$a^T_{\ell m} \propto \int d^3k \;\; Y^*_{\ell m}(\hat{k})\, T^\zeta_T(k) \left[\zeta(\vec{k}) + (-1)^\ell \zeta(-\vec{k})\right] ,$$

$$a^B_{\ell m} \propto \int d^3k \;\; {}_{-2}Y^*_{\ell m}(\hat{k})\, T^\gamma_B(k) \left[\gamma_R(\vec{k}) - (-1)^\ell \gamma_L(-\vec{k})\right] ,$$

$$\langle a^B_{\ell_1 m_1} a^B_{\ell_2 m_2} a^T_{\ell_3 m_3}\rangle \sim \left[\langle \gamma_R \gamma_R \zeta\rangle + (-1)^{\ell_1+\ell_2+\ell_3} \langle \gamma_L \gamma_L \zeta\rangle\right] .$$

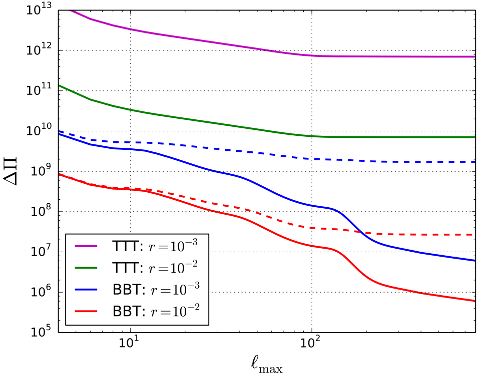
- For the multipole location $\ell_1 + \ell_2 + \ell_3 =$ odd we have:

$$\langle a^B_{\ell_1 m_1} a^B_{\ell_2 m_2} a^T_{\ell_3 m_3}\rangle \sim [\langle \gamma_R \gamma_R \zeta\rangle - \langle \gamma_L \gamma_L \zeta\rangle] \propto \Pi \, .$$

- Constraint over *BBT* $\Longrightarrow$ Constraint over Π.
- We can also use the *TTT* CMB bispectrum, but with worse S/N.

Fisher matrix forecast in preparation

- *"Measuring chiral gravitational waves in Chern-Simons gravity with CMB bispectra"*, M. Shiraishi, G. Orlando, N. Bartolo.



- In Fig. expected 1σ errors on Π for an ideal experiment are shown. Taking $\ell_{max} \approx 500$ and $r = 10^{-2}$ we have $\Delta\Pi_{min} \approx 10^6$.

Need for time dependent Chern-Simons mass

- Neglecting time-dependence of M_{CS} during inflation is equivalent to

$$\frac{\dot{M}_{CS}}{HM_{CS}} = \epsilon - \eta - \sqrt{2\epsilon} M_{Pl} \frac{f''(\phi)}{f'(\phi)} \lesssim 1 \, .$$

- This condition, given $r \sim 10^{-2}$, forces Π to be smaller than 10^2 (impossible to observe).
- To measure Π we need M_{CS} to vary significantly during inflation. In particular, we need M_{CS} to increase during inflation, important to avoid ghost formation.
- A large variation of M_{CS} during inflation can drastically enhance Π! Possibilities for a detection with CMB bispectra.

Squeezed modulation from an external field (ongoing)

- An external scalar field χ interacting with Chern-Simons gravitational term can provide a squeezed modulation of GW power spectrum.

Squeezed modulation of the tensor power spectra

$$\Delta_\gamma^{R/L}(q)|_\chi = \Delta_\gamma^{R/L}(q)|_{(0)} + \chi(Q)\frac{\langle\gamma_{R/L}(q_1)\gamma_{R/L}(q_2)\chi(Q)\rangle'}{P_\chi(Q)},$$

$$Q \to 0\,, \qquad q_1 \simeq q_2 = q\,.$$

- R and L squeezed bispectra differ for a minus sign $\rightarrow$ Enhancement of the power spectrum chirality!
- The modulation can be scale dependent. Possibility to enhance chirality at interferometer frequencies.

Summary and Conclusion

- We focused on the Chern-Simons gravitational term coupled to the inflaton field during inflation.
- Effects on power-spectrum chirality are killed if we work in a regime free from ghost formation. Impossible to observe it using CMB power-spectra.
- Chirality of the bispectrum $\langle \gamma\gamma\zeta \rangle$ can be enhanced if we admit a rapidly increasing M_{CS} during inflation. Possibility of a detection using *BBT* bispectrum.
- Ongoing: study the effects of an external scalar field χ coupled to gravitational Chern-Simons term during inflation. Interesting for enhancing chirality at interferometer scales.