On Axions in Inflation and as Hot Relics

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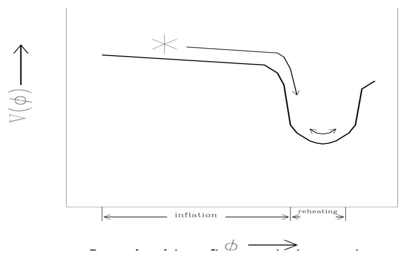
talk @ ASI/COSMOS, Early Universe meeting (Padova).

[1] In collaboration with Ricardo Z.Ferreira

Slow-roll



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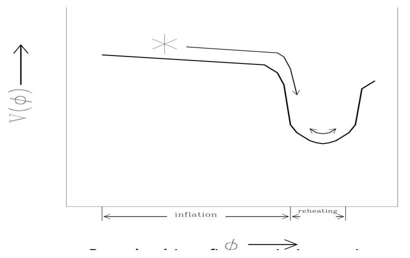

- "Slow-roll" (no particles, “vacuum")
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- Vacuum Density fluctuations

Slow-roll Inflation: simple but...

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- Why unusually flat $V(\phi)$?

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- Interesting to explore non-minimal features during inflation:
  - Different predictions?
  - Maybe even new dynamics?

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- We analyze:
  - Axion models
  - $\rightarrow$ Dissipation & Temperature, *already during inflation*
  - $\rightarrow$ Strong Friction during inflation:
    - Can we get rid of flat potentials?
    - Phenomenologically viable?

Inflation with Axial coupling

Inflaton (ϕ) coupled to U(1) gauge fields, “axion-like":

$$L = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi) + \frac{\phi}{4f_\gamma}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

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Moreover during inflation:

- $F_{\mu\nu}\tilde{F}^{\mu\nu}$ odd under CP (and so T) $\Longrightarrow$ Instability during inflation $\propto \dot{\phi}$!

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- When $\phi' = a\dot{\phi} \neq 0 \rightarrow$ One helicity is unstable: gauge fields becomes quickly large

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Constant $\dot{\phi}$ and de Sitter

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- Solution at $\tau \to 0^-$ (future):

$$A_+ \approx \frac{1}{\sqrt{2k}} \left(\frac{k|\tau|}{2\xi}\right)^{1/4} e^{\pi\xi - 2\sqrt{2\xi k|\tau|}}$$

Consequences

- (Sorbo & Anber '09) estimated:

$$\frac{\langle F\tilde{F}\rangle}{4} = \frac{1}{2a^4}\int \frac{d^3k}{(2\pi)^3}\, k\, \frac{d\left[|A_+|^2 - |A_-|^2\right]}{d\tau} \approx \frac{H^4}{\xi^4} e^{2\pi\xi}$$

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  - At large ξ: Backreaction on ϕ dynamics (friction)

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- Moreover loop expansion [5] seems to break down at $\xi \gtrsim 3.5 - 4.5$

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  - Very efficient if Standard Model gauge fields: $\gamma\gamma \leftrightarrow \ell^+\ell^-$

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  - Perhaps: oscillations in spectra?

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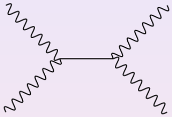
$$\begin{cases} N_\gamma(k) \simeq \quad 0, & k/a \gg H \\ N_\gamma(k) \simeq \frac{e^{2\pi\xi}}{8\pi\xi}, & k/a \ll H \end{cases}$$

Scatterings

Scatterings are enhanced by powers of N_γ



$$\frac{dN_\gamma(k)}{d\tau} = S(k)$$

$$S = \frac{1}{\omega(k)} \int \prod_{i=2}^{4} \left(\frac{d^3 p_i}{(2\pi)^3 (2E_i)} \right) |M_n|^2 (2\pi)^4 \delta^{(4)} \left(k^\mu + p_2^\mu - p_3^\mu - p_4^\mu \right) \cdot$$

$$\cdot N_\gamma(k) N_\gamma(p_2) \left[1 + N_\gamma(p_3) \right] \left[1 + N_\gamma(p_4) \right]$$

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- Scattering rates $S \approx 10^{-4} \frac{\omega^5}{f^4} N_+^3 \Longrightarrow$ For large N_γ:

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- Expectation: thermal bath of photons with temperature

$$T \approx \rho_{\gamma,\rm initial}^{1/4} \approx 0.1 H e^{\pi\xi/2}$$

Boltzmann-like equations

- We rewrite the eom as a Boltzmann-like eq. ($g \equiv A'_+(k,\tau)/A_+(k,\tau)$) :

$$\frac{dN_{\gamma_+}(k)}{d\tau} = -\frac{4k\xi}{\tau}\frac{\mathrm{Re}\,[g]}{|g|^2+k^2}\left(N_{\gamma_+}(k)+\frac{1}{2}\right)$$

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- Full system (A_+, A_-, ϕ)

$$\begin{cases} N'_+ = -\frac{4k\xi}{\tau}\frac{\mathrm{Re}[g]}{|g|^2+k^2}\left(N_+ + \frac{1}{2}\right) + S^{++} + S^{+\phi} + D^{+\phi} + S^{+-}\,, \\ N'_\phi = -S^{+\phi} - D^{+\phi}\,, \\ N'_- = -S^{+-}\,, \end{cases}$$

Numerical results

- Particle distributions approaches Bose-Einstein distribution at ξ, f in agreement with estimations.

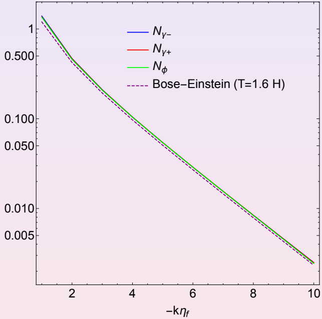
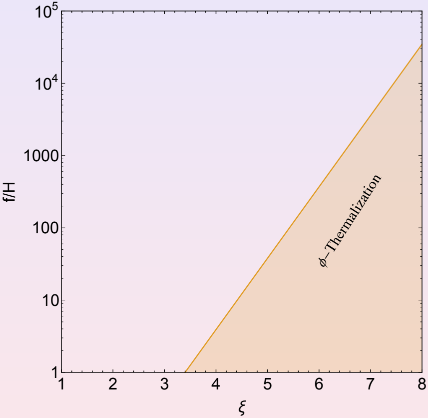


Figure: $\xi = 2$, $f/H = 0.1$

Duration of simulation: $\mathcal{O}(1)$ e-fold, $\{\tau_0 = -2, \tau_f = -1\}$

Discretize: $\mathcal{O}(10)$ modes of comoving momentum $\boxed{H \lesssim k/a_0 \lesssim \mathcal{O}(10)H}$.

Thermalization



$$\Gamma_s \gg H \qquad \Longrightarrow \qquad \xi \gtrsim 0.45 \ln\left(\frac{f}{H}\right) + 2.7\,,$$

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- Using $\sigma_{\gamma\gamma\leftrightarrow\ell^+\ell^-} \approx \frac{\alpha_{EM}^2}{\omega^2}$

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- More efficient, interactions not suppressed by powers of $1/f$.
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- Using $\sigma_{\gamma\gamma\leftrightarrow\ell^+\ell^-} \approx \frac{\alpha_{EM}^2}{\omega^2} \implies$

  Requirement for thermalization ($\Gamma_s \gg H$) :

$$N_\gamma H \ll \frac{\alpha_{EM}^2}{\omega^2} \cdot H^3 N_\gamma^2 \quad \implies \quad \boxed{\xi \gtrsim 2.9}$$

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Phenomenology in the thermal regime

- Loop effects on ζ drastically modified!
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  - Constraints on ξ become weaker and (maybe) allow for the backreacting regime? (Work in progress)

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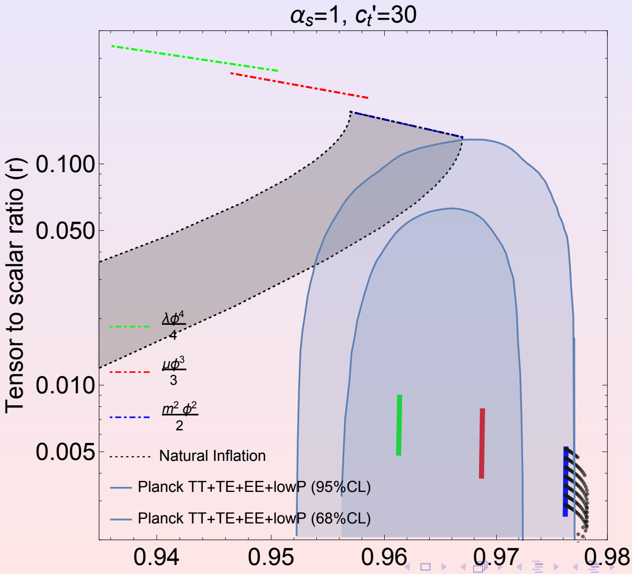
- At least $\mathcal{O}(10^{-2})$ suppression.
- $\Longrightarrow$ Large field potentials can be viable again

Thermalized Large Field Potentials



Axions in Inflation and as Hot Relics

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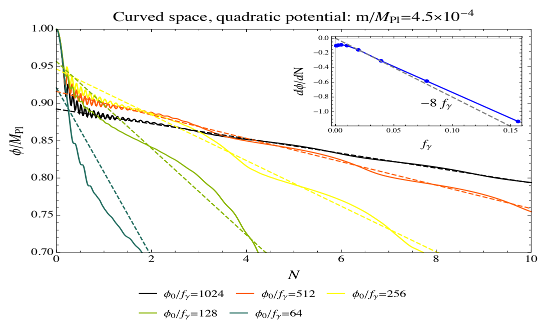

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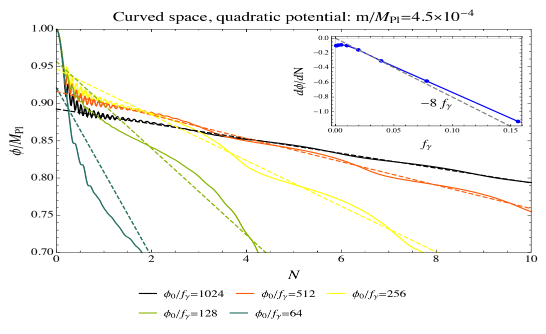

- Period of about 4 efolds, Amplitude $\propto f$

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- Prediction: larger $N_{\rm eff}$.

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