The Cosmic Microwave Background Bispectrum

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Primordial non-Gaussianity

The non-Gaussianity (NG) of the primordial perturbations field $\Phi(k)$ is a powerful probe of early Universe Physics.

- The statistic most sensitive to the NG component is the three-point function, the Bispectrum.

$$B_\Phi(k_1, k_2, k_3) = \langle \Phi(k_1)\Phi(k_2)\Phi(k_3) \rangle$$

- The CMB Angular Bispectrum is linearly linked to the primordial one via the radiation transfer function $\Delta(k)$.

$$B_{\ell_1 \ell_2 \ell_3}^{m_1 m_2 m_3} = \langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle$$

Bispectrum Estimation

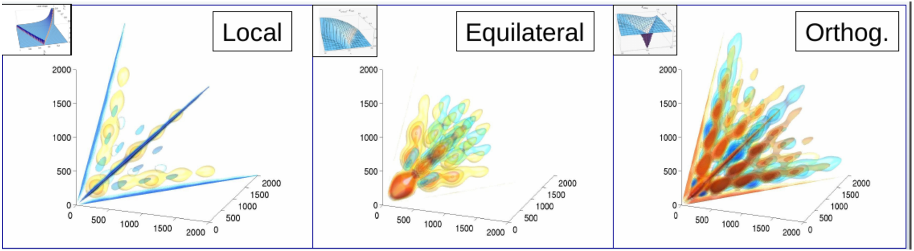
The measurement of CMB Bispectrum presents two main issues:

- The S/N of a single configuration is too small to be detected.
- The computational cost is very high $\mathcal{O}(\ell^5)$.

- The NG sygnal is parametrized by the overal NG amplitude f_{NL}.
- The amplitude is estimated fitting theoretically motivated template to the data.
- To reduce the computational cost the template is written in separable form on the three wavenumbers:

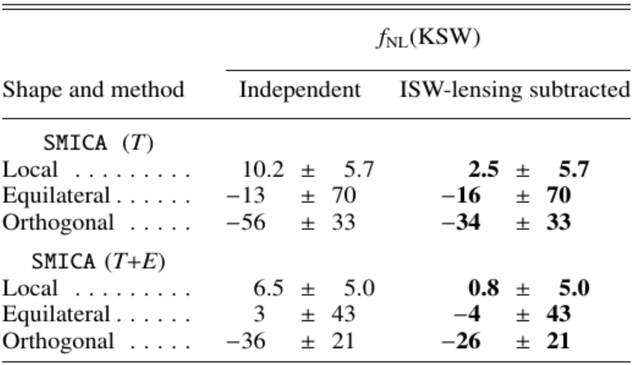
$$B^{m_1 m_2 m_3}_{\ell_1 \ell_2 \ell_3} = X_{\ell_1} Y_{\ell_2} Z_{\ell_3} + permutations.$$

Shapes



- **Local** : Multifield Inflation, Ekpyrotic models
- **Equilateral/Orthogonal**: single-field models with non-standard kinetic terms or higher derivative terms, Effective Field Theory
- **Flat** (Equil. + Ortho.): non Bunch Davies vacuum

Planck constraints and prospects

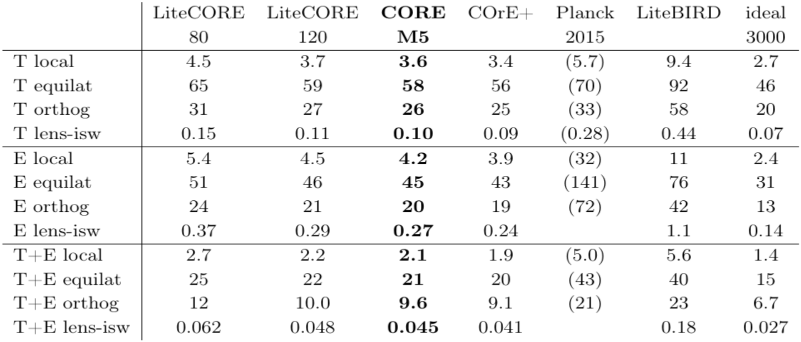


	$f_{\rm NL}$(KSW)	
Shape and method	Independent	ISW-lensing subtracted
SMICA (T)		
Local	10.2 ± 5.7	**2.5 ± 5.7**
Equilateral	−13 ± 70	**−16 ± 70**
Orthogonal	−56 ± 33	**−34 ± 33**
SMICA $(T{+}E)$		
Local	6.5 ± 5.0	**0.8 ± 5.0**
Equilateral	3 ± 43	**−4 ± 43**
Orthogonal	−36 ± 21	**−26 ± 21**

(Planck Collaboration, 2015 - arXiv:1502.01592)

- Planck provides constraints on f_{NL} for these shapes very close to the theoretical limit for CMB observations
- Future surveys could still improve the sensitivity by a factor ~ 2.

Planck constraints and prospects



	LiteCORE 80	LiteCORE 120	**CORE** **M5**	COrE+	Planck 2015	LiteBIRD	ideal 3000
T local	4.5	3.7	**3.6**	3.4	(5.7)	9.4	2.7
T equilat	65	59	**58**	56	(70)	92	46
T orthog	31	27	**26**	25	(33)	58	20
T lens-isw	0.15	0.11	**0.10**	0.09	(0.28)	0.44	0.07
E local	5.4	4.5	**4.2**	3.9	(32)	11	2.4
E equilat	51	46	**45**	43	(141)	76	31
E orthog	24	21	**20**	19	(72)	42	13
E lens-isw	0.37	0.29	**0.27**	0.24		1.1	0.14
T+E local	2.7	2.2	**2.1**	1.9	(5.0)	5.6	1.4
T+E equilat	25	22	**21**	20	(43)	40	15
T+E orthog	12	10.0	**9.6**	9.1	(21)	23	6.7
T+E lens-isw	0.062	0.048	**0.045**	0.041		0.18	0.027

(CORE Collaboration, 2016 - arXiv:1612.08270)

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Scale Dependent Bispectra

A scale dependent (SD) $f_{\rm NL}$ is a natural prediction of many inflationary models. Measuring the Bispectrum slope $n_{\rm NG}$ can provide new information about Inflation.

"Local" SD templates

Multifield models: curvaton, modulated reheating; mixed inflaton-curvaton.

- $B^{1f}_{\Phi}(k_1, k_2, k_3) \propto f_{\rm NL} \left[(k_1 k_2)^{n_\zeta - 4} k_3^{n_{\rm NG}} + 2 \text{ perms.}\right]$
- $B^{2f}_{\Phi}(k_1, k_2, k_3) \propto f_{\rm NL} \left[(k_1 k_2)^{n_\zeta + (n_{\rm NG}/2) - 4} + 2 \text{ perms.}\right]$

"Equilateral" SD templates

Single-field models with non canonical kynetic terms (DBI-Inflation)

- $B^{gm}_{\Phi}(k_1, k_2, k_3) \propto f_{\rm NL} \, (k_1 k_2 k_3)^{n_{\rm NG}/3} \, B^{equil.}_{\Phi}(k_1, k_2, k_3)$

Scale Dependent Bispectra

Estimator

1. We use an extension of the standard KSW estimator, including the scale dependent shapes:

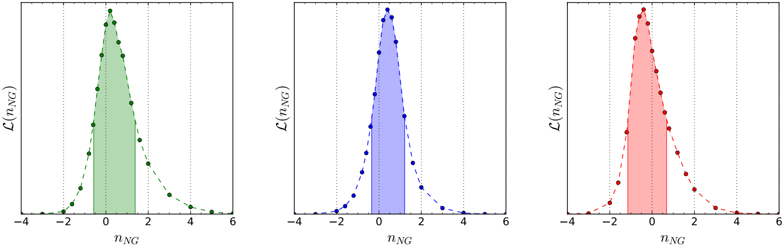
$$\hat{f}_{NL}(n_{\mathrm{NG}}) = \frac{1}{\mathcal{N}} \sum \frac{B^{th}_{\ell_1\ell_2\ell_3}(f_{\mathrm{NL}} = 1, n_{\mathrm{NG}}) B^{obs}_{\ell_1\ell_2\ell_3}}{C_{\ell_1} C_{\ell_2} C_{\ell_3}}$$

2. We iteratively obtain estimates of $f_{NL}(n_{\mathrm{NG}})$ for different values of the running.
3. We use these value to interpolate the marginalized Likelihood $\mathcal{L}(n_{\mathrm{NG}}) \propto exp(\hat{f}^2_{\mathrm{NL}})$.
4. The value of n_{NG} is then directly derived from the Likelihood.

Scale Dependent Bispectra

WMAP9 analysis

one-field model - two-fields model - geometric mean model



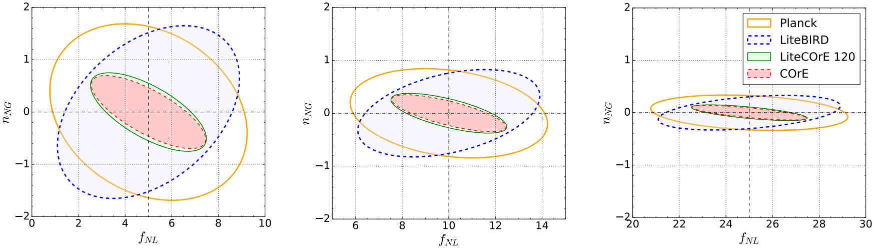
model	$n_{\rm NG}$	$\mathbf{k}_{piv}$
one-field (local)	$0.2^{+1.2}_{-0.8}$	$0.035\,\mathrm{Mpc}^{-1}$
two-fields (local)	$0.4^{+0.8}_{-0.7}$	$0.01\,\mathrm{Mpc}^{-1}$
geometric mean (equil.)	$-0.4^{+1.0}_{-0.7}$	$0.01\,\mathrm{Mpc}^{-1}$

(Oppizzi et al., 2017 - arXiv:1711.08286)

Scale Dependent Bispectra

Forecasts

Expected sensitivity for different amplitude values for the local one-field model.



Experiment	(ℓ_{max})	$f_{\rm NL} = 5$	$f_{\rm NL} = 10$	$f_{\rm NL} = 25$
Planck	(2400)	1.7	0.8	0.3
LiteBIRD	(1350)	1.6	0.8	0.3
LiteCOrE 120	(3000)	0.7	0.4	0.1
COrE	(3000)	0.7	0.3	0.1

Bispectrum from Scalar-Scalar-Tensor Correlations

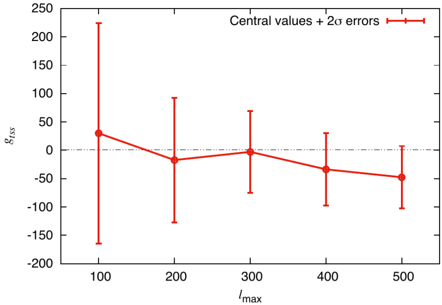
- Sourced by nonlinear coupling between one Graviton and two scalars.
- Peaked on squeezed configurations, but uncorrelated with the standard local shape due to differences in the oscillatory pattern.
- Has a very complex form and cannot be factorized.

Modal methodology: the bispectrum is decomposed into a product of separable eigenfunctions.

Central value and 2σ errors on g_{tss} from WMAP9, as a function of ℓ_{max}.

$$g_{tss} = -48 \pm 28$$

(Shiraishi et al. , 2017 - arXiv:1710.06778)



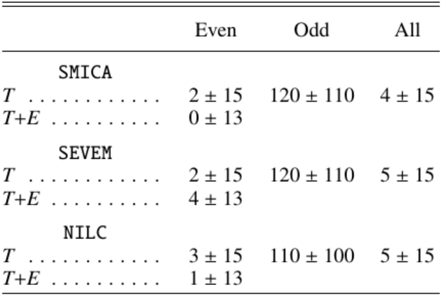
Parity-Odd Bispectra

- Under parity, the allowed Bispectrum configurations are restricted to $\ell_1 + \ell_2 + \ell_3 = \textit{even}$
- Some Early Universe scenarios predict parity-odd tesor NG (Weyl gravity, rolling pseudoscalar, helical primordial magnetic field model)
- $\ell_1 + \ell_2 + \ell_3 = \textit{odd}$ configurations are generated in these models. Standard estimators are blind to this component.

The optimal estimator include even and odd component

$$\hat{f}^{all}_{\mathrm{NL}} = \frac{N^{even}\,\hat{f}^{even}_{\mathrm{NL}} + N^{odd}\,\hat{f}^{odd}_{\mathrm{NL}}}{N^{even} + N^{odd}}$$

Not separable shape: modal estimator



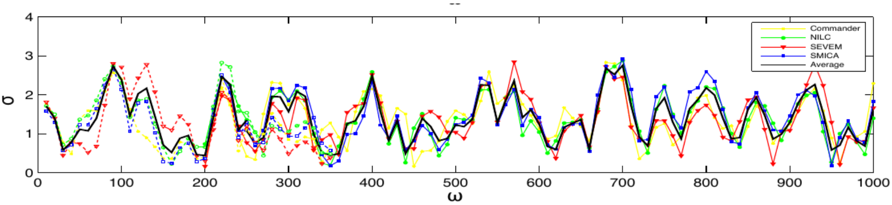
	Even	Odd	All
SMICA			
T	2 ± 15	120 ± 110	4 ± 15
T+E	0 ± 13		
SEVEM			
T	2 ± 15	120 ± 110	5 ± 15
T+E	4 ± 13		
NILC			
T	3 ± 15	110 ± 100	5 ± 15
T+E	1 ± 13		

Oscillatory Bispectra

A number of Bispectrum models present oscillatory features different from the standard shapes.

- Feature models: $B^{feat} \propto \sin[\omega(k_1 + k_2 + k_3) + \phi]$
- Resonance models: $B^{res}_{\Phi} \propto \sin[C \ln(k_1 + k_2 + k_3) + \phi]$

- Testing this models implies to build a frequency dependent estimator and to scan the frequency space searching for significant peak.
- Results shall be interpreted accounting for the "look elsewere" effect.
- Planck provides "hints" of a NG signal for equilateral and flattened oscillatory shapes spread across several broad peaks.



Conclusion

- The CMB Bispectrum is a powerful benchmark for Early Universe Theories.
- Measuring Bispectra is a challenging task: ad-hoc estimators shall be developed for each model.
- Current constraints almost saturate the sensitivity for the standard shapes.
- Many Bispectrum template still remain to be tested:
    - Scale dependent bispectra
    - Parity-odd bispectra
    - Scalar-scalar-tensor correlations
    - Oscillatory templates